Students' Ways of Knowing and Learning Mathematics and Their Ways of Interacting with Advanced Calculators

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In this paper, the relationships between students' beliefs about knowing and learning mathematics, and how they engage with calculators, are investigated. An online survey was conducted for 964 Singaporean and 176 Victorian senior secondary students. Students' *connected knowing–deep approach* conception of mathematics was found to be associated with their use of *calculators as collaborator*, and their *separate knowing–surface approach* conception of mathematics was associated with use of *calculators as master*. Gender differences in students' beliefs were also found.

From past research, students' conceptions about mathematics are found to be related to their learning approaches and their academic achievement, and influenced by the learning contexts they experience (Muis, 2004). Since the inclusion of technology in mathematics education, particularly the use of graphic calculators (GC) and calculators with computer algebra systems (CAS), there have been few studies in which the relationships between students' conceptions of mathematics, approaches to learning mathematics and how they interact with such technologies have been investigated. The aim of this study was to address this gap in the literature.

Background Literature

There are many studies on students' epistemological beliefs (beliefs about knowledge and ways of knowing) both in general (e.g., Hofer & Pintrich, 1997), and about mathematics (e.g., Muis, 2004; Roesken, Pepin, & Toerner, 2011). There are many theories in both fields, but no unified framework exists; mathematics education researchers have generally investigated beliefs about mathematics as a construct under the affective domain (Muis, 2004). The interest in affect and beliefs in mathematics started in the 1980s from investigations of teachers' beliefs which influenced their instructional approaches and it has since blossomed into a field encompassing a multitude of theories and views about the various aspects of beliefs and belief systems (Roesken, Pepin, & Toerner, 2011). In a review of research on epistemological beliefs, Muis (2004) found that students generally believed in an innate ability to do mathematics and considered knowledge to be unchanging, composed of unrelated components, and handed down by an authority figure.

Students' beliefs have also been found to be related to their learning strategies; for example a belief in mathematics as a list of isolated facts and fixed procedures might encourage students to concentrate on memorising lists and procedures as a study strategy (Muis, 2004; Schommer-Aikins & Easter, 2006). Although there were significant associations between students' epistemic beliefs about mathematics and the types of behaviours students engaged in when learning mathematics, no strong evidence of causality was identified (Muis, 2004). Muis (2004) suggested that there could be other types of relationships between beliefs and learning strategies, such as a reciprocal relationship.

As technology becomes more integrated into the mathematics curricula in different countries, there are many studies conducted which investigate how students interact with the advanced calculators and their learning outcomes (e.g. Burrill et al., 2002). Students' use of

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calculators generally resulted in positive attitudes and improvement of learning, and was found to be influenced by factors such as the role of calculators in assessment and instruction, and students' familiarity with and attitude towards the tool. Students tended to use calculators for quick and accurate graphing, and for problem solving and investigations; however evidence suggests that calculators might be underutilised (Burrill et al., 2002). There were mixed results regarding gender differences in students' use of calculators, with some studies showing no evidence of gender difference, and others with a gender difference favouring females (Burrill et al., 2002) or males (e.g. Tan & Forgasz, 2011).

Even though students' beliefs and attitudes are generally acknowledged as influenced by socio-cultural contexts, there were few cross-country comparisons (Roesken, Pepin, & Toerner, 2011). There does not appear to be research studies in which the relationships between students' gender, beliefs, learning strategies and their engagement with technologies have been investigated. It is thus of interest in the current study to investigate students' mathematical beliefs and learning approaches in technology-rich environments where advanced calculators are used. Two research questions are the focus of this report which formed part of a larger study:

1. Are there any differences in students' ways of knowing and approaches to learning mathematics (a) by region, (b) by gender within each region?
2. Are there any relationships between students' ways of knowing, approaches to learning mathematics, and their ways of interacting with the advanced calculators, (a) by region, (b) by gender within each region?

The Study

Theories Underpinning the Study

Students' ways of knowing. Becker (1995) applied the *Women's ways of knowing* model by Belenky, Clinchy, Goldberger and Tarule (1986) to mathematics. She described two types of knowing, Separate and Connected. *Separate Knowing* (SK) deals with "logic, rigour, abstraction, rationality, axiomatics, certainty, deduction, completeness, absolute truth, power and control, algorithmic approach, structure and formality" (Becker, 1995, p. 167). *Connected Knowing* (CK) deals with "intuition, creativity, hypothesizing, conjecture, experience, relativism, induction, incompleteness, personal process tied to cultural environment, contextual" (Becker, 1995, p. 167). These ways of knowing have been found to be gender-related, with males tending to score higher in separate knowing than females (e.g., Schommer-Aikins & Easter, 2006).

Students' approaches to learning. Building on the early work of Marton and Säljö (1976), Biggs (1987) described students' learning approaches to be composed of two components: motive – the intention to learn, and strategy – the way to go about learning. A *Deep Approach* (DA) is intrinsic and meaningful, where students "study to actualize interest and competence" (Biggs, 1987, p. 11) through seeking understanding and relating to ideas. A *Surface Approach* (SA) is instrumental and reproductive, where the "main purpose is to meet requirements minimally... (and to) limit target to bare essentials and reproduce (content) through rote learning" (Biggs, 1987, p. 11). It was found that students who thought of mathematics as a fragmented body of knowledge tended to learn it using surface approaches, whereas students who viewed mathematics as an interconnected complex system tended to use deep approaches (Crawford, Gordon, Nicholas, & Prosser, 1994). Findings on gender differences are ambivalent. Gordon (1997), for example, found that females score higher than males on surface approaches, while Kılıç and Sağlam (2010) found that males scored higher on rote learning (surface approach) and females scored higher on meaningful learning (deep approach).

Students' ways of interacting with technology. In order to describe the ways teachers and students interact with technology for teaching and learning mathematics, Goos, Galbraith, Renshaw and Geiger (2000) proposed four metaphors for the role of technology that comprise the Master, Servant, Partner, and Extension of self (MSPE) framework:

- Master (M), where one is subservient to technology (e.g., blindly follow calculator steps in the textbook without understanding);
- Servant (S), where one uses technology for menial or tedious tasks (e.g., using calculators to replace mental or pen-paper calculations);
- Partner (P), where one treats technology as an equal that provides resources and information (e.g., using calculators as a platform for collaborative inquiry); and
- Extension of Self (E), where one engages with the technology intimately and seamlessly (e.g., integrating calculators appropriately in mathematical explorations as part of one's intellectual repertoire of mathematical tools and skills).

Tan (2009) developed an instrument based on the MSPE framework and administered it to Singaporean and Victorian senior secondary mathematics students. Factor analysis revealed three, rather than four, factors. The technology as Partner and Extension of Self items loaded as one factor that was named technology as Collaborator (Tan & Forgasz, 2011). In this paper, further findings from the same study are reported. The focus is on the relationships between students' ways of knowing and approaches to learning mathematics, and their ways of interacting with advanced calculators.

Instruments, Sample, and Data Analysis

The main study was conducted in Singapore (on GC) and Victoria (on CAS calculators) using an online survey. Invitations were sent to students through schools in both regions to participate in the study. Additionally an advertisement was put up in Facebook to recruit student participants from Victoria (Tan & Forgasz, 2011).

Instruments. The relevance, applicability, and length of pre-existing instruments associated with the various theoretical frameworks described above were closely considered in the development of the instruments used in the present study. Some modifications were needed.

1. *Measuring students' ways of interacting with technology.* Students' ways of interacting with technology (GC and CAS calculators) were measured using a researcher developed instrument adapted from the MSPE framework. Following a pilot of the instrument with multiple items, the final instrument was comprised of 12 items measuring Calculator as Master (Cal_Ma), as Servant (Cal_Se), and as Collaborator (Cal_Co) - see Tan (2009) for more details.
2. *Measuring ways of knowing and learning mathematics.* Students' separate (SK) and connected knowing (CK) were measured using a researcher developed instrument adapted from Ocean's (1998) survey items. Students' approaches to learning (surface [SA] and deep [DA]) were measured by adapting the 22 items from Biggs' Learning Process Questionnaire (Kember, Biggs, & Leung, 2004) to mathematics. A 5-point Likert type format (1=strongly disagree, 5=strongly agree) was used for consistency.

Analysis of data from a pilot study of 189 Singaporean students showed that there were moderate to strong positive correlations between the CK and DA scores (r=0.606) and between SK and SA scores (r=0.401). Since the close relationship between students' beliefs and their study behaviours found in this data analysis is also found in the literature, it was decided to pool the items from the two instruments to increase the reliability of the subscales and shorten the length of the instrument. An iterative process of analysing reliability tests and principal component analyses (PCA) was employed to eliminate poorly performing items. The PCA confirmed the existence of two factors, with seven items from connected knowing and deep approach forming one factor, and seven items from separate knowing and surface approach forming another. The combined instrument, *Ways of Knowing and Learning Mathematics*, measured *Connected Knowing–Deep Approach* (CK-DA) and *Separate Knowing–Surface Approach* (SK-SA) (see Table 1) and had Cronbach-α reliability measures within acceptable limits ($\alpha_{\text{CK-DA}}$ =0.792; $\alpha_{\text{SK-SA}}$=0.683). This new scale was used in the main study.

Table 1
Items in 'Ways of Knowing and Learning Mathematics' Instrument by Construct

Connected Knowing and Deep Approach (CK-DA)	Separate Knowing and Surface Approach (SK-SA)
CK	SK
Maths makes you think creatively	In maths, something is either right or it is wrong.
Good maths teachers show students several different ways to look at the same question.	To solve maths problems you have to be taught the right procedure or you can't do anything.
In maths you can be creative and discover things for yourself.	When I solve maths problems, I'm often stuck if I can't remember the next step.
DA	SA
I try to relate what I have learned in maths to what I learn in other subjects.	When I score poorly on a maths test, I worry a lot about how I will do on the next one.
I work hard at my studies because I find mathematics interesting.	I see no point in learning material which is not likely to be in the examination.
As I am reading, I try to relate new concepts and ideas to what I already know about that topic.	Even when I have studied hard for a maths test, I worry that I may not be able to do well in it.
I frequently think about how to solve maths problems even while on the bus or lying on my bed.	I learn maths formulas by heart even if I don't understand them.

Sample. There were 964 Singaporean students (37.1% males, 62.9% females), and 176 Victorian students (31.3% males, 68.8% females).

Data analysis. Data were collected and analysed using Statistical Package for Social Sciences 18.0 (SPSS) software. T-tests and Mann-Whitney U tests were used to compare regional and gender differences. Pearson product-moment correlation coefficients were used to analyse the relationships between the variables.

Results and Discussion

Factor analysis of the 14 items in the Ways of Knowing and Learning Mathematics revealed similar results to the pilot study: the seven CK and DA items formed one component, and the seven SK and SA items the other (see Table 1). Cronbach–α values for the two subscales by region were: CK-DA ($\alpha_{S'pore}$=0.774; α_{Vic}=0.798), and SK-SA ($\alpha_{S'pore}$=0.693; α_{Vic}=0.625); all were reasonable (Pallant, 2007).

Regional and Gender Differences in Ways of Knowing and Learning Mathematics

Mean subscale scores were calculated and reduced to the range 1 to 5 for ease of interpretation. The results of comparisons of the mean subscale scores by region, and by gender within region, using t-tests and Mann-Whitney U tests are shown in Table 2.

Table 2

Regional and Gender Comparisons: N, Mean Scores, Test Statistics, and Effect Sizes

		Singapore				Victoria			
Factor	Gender	N	Mean	Test statistic	Effect size	N	Mean	Test statistic[1]	Effect size
CK-DA	Overall	928	3.28			125	3.34		
	Female	586	3.21	–3.67, p<0.001	0.15	100	3.27	2.04, p<0.05	0.18
	Male	342	3.39			25	3.61		
SK-SA	Overall	933	3.47			124	3.50		
	Female	587	3.51	2.46, p<0.05	0.08	100	3.55	–2.29, p<0.05	0.21
	Male	346	3.40			24	3.28		

[1] Mann- Whitney U test was used for Victorian data due to small sample size.

Interestingly, there were no significant regional differences in the CK-DA and SK-SA scores of Singaporean and Victorian students. Students in the two regions have comparable results on connected knowing-deep approach and separate knowing-separate approach. Also, the mean scores for SK-SA (S'pore=3.47; Vic=3.50) were higher than that for CK-DA in both regions (S'pore=3.28; Vic=3.34), with the difference being significant only for the Singaporeans: $t(921)= –5.362$, p<0.001. This implies that students agree more strongly with having a separate knowing-surface approach than to having a connected knowing-deep approach to the learning of mathematics. Since both the Singaporean and the Victorian senior secondary mathematics curricula culminate in high stake examinations, this finding of high SK-SA scores is consistent with the conclusion from other studies that argue that an extreme emphasis on academic performance and examination success is associated with a tendency to adopt a surface learning approach (e.g., Kılıç & Sağlam, 2010).

As can be seen from Table 2, there are clear gender differences in students' ways of knowing and learning mathematics which are common to both regions. On average, males scored higher than females in CK-DA, whereas females had higher SK-DA scores than males; the Victorian data had higher effect sizes than the Singaporean data. Overall, these findings suggest that males were more likely to hold connected knowing conceptions about mathematics and use deep approaches to learn mathematics, whereas females were more likely to hold separate knowing conceptions and use surface approaches. This finding is consistent with Gordon's (1997) result that females were more likely to employ a surface approach to mathematics learning. However, the finding that males were more likely than females to employ connected knowing is different from what the gender literature suggests (Becker, 1995).

Relationships between Ways of Knowing and Learning Mathematics and Ways of Interacting with Technology

The correlation coefficients between students' CK-DA, SK-SA and their ways of interacting with calculators are found in Table 3.

Table 3
Correlations between CK-DA, SK-SA, and Calculator as Master, Servant and Collaborator

			Cal _ Ma	Cal _ Se	Cal _Co
CK-DA	S’pore	Overall	–0.069*	–0.094**	0.412**
		Female	–0.028	–0.029	0.384**
		Male	–0.101	–0.202**	0.426**
	Vic	Overall	–0.230*	–0.108	0.405**
		Female	–0.199	–0.053	0.366**
		Male	–0.118	–0.222	0.520*
SK-SA	S’pore	Overall	0.362**	0.213**	–0.095**
		Female	0.351**	0.240**	–0.117**
		Male	0.368**	0.183**	–0.043
	Vic	Overall	0.519**	0.342**	0.007
		Female	0.478**	0.335**	–0.028
		Male	0.614**	0.148	0.220

* $p < 0.05$. ** $p < 0.01$. *** $p < 0.001$.

It can be seen in Table 3 that common associations are evident in both regions:

- Using calculator as Master (Cal_Ma), i.e., being subservient to technology, is positively associated with having a Separate Knowing and Surface Approach towards mathematics learning ($r_{S’pore}$=0.362; r_{Vic}=0.519). Students who learnt mathematics by memorising the steps without real understanding also tended to have a technological or mathematical dependence on the calculators.
- Using calculator as Collaborator (Cal_Co), i.e., engaging with the calculator as a Partner and as an Extension of Self, is positively associated with having a Connected Knowing and Deep Approach towards mathematics learning ($r_{S’pore}$=0.412; r_{Vic}=0.405). Students who learnt mathematics through intrinsic interest and deep understanding also tended to use the calculators at a more sophisticated level for problem solving and mathematical exploration.
- Using calculator as Servant (Cal_Se), i.e., using calculators for tedious, repetitive computation and to replace pen-paper calculations, has a weak to moderate positive association with Separate Knowing and Surface Approach ($r_{S’pore}$=0.213; r_{Vic}=0.342). Students who held a rigid conception of mathematics and use a rote learning

approach to mathematics learning are also likely to use the calculators for menial tasks.

Using a Fisher transformation to compare the correlation coefficients between the two regions, only one significant regional difference was found: between SK-SA and Cal_Ma ($z = -1.98$, $p<0.05$). The correlation between SK-SA and Cal_Ma was stronger for the Victorian ($r=0.519$) than the Singaporean sample ($r=0.362$). Interestingly, scatterplots of the SK-SA and Cal_Ma scores for both regions revealed that there were higher proportions of Singaporean than Victorian students who had low SK-SA scores and high Cal_Ma scores. This group of Singaporean students did not hold Separate Knowing conceptions or use Surface Approaches, yet used their calculators as Masters. This finding was consistent with the finding reported in an earlier paper that Singaporeans were less fluent in using GCs than Victorians were in using CAS calculators (Tan & Forgasz, 2011).

It can be seen in Table 3 that the coefficients were generally comparable across gender. Analysis using a Fisher transformation resulted in only one significant gender difference: the correlation between CK-DA and Cal_Se in the Singaporean sample. For Singaporean females there was no association between CK-DA and Cal_Se, whereas for males there was a weak negative correlation. Since these correlations were absent or weak, the difference was not considered to be educationally significant.

Conclusion

In summary, students' ways of interacting with advanced calculators are found to be associated with their beliefs about mathematics and approaches to mathematics learning. The use of calculators as Master (being subservient to technology) is associated with high Separate Knowing-Surface Approach, and the use of calculators as Collaborator (high level of sophistication) is associated with high Connected Knowing-Deep Approach. These associations were common across the two regions in this study and gender. Although causality cannot be confirmed, an implication for teaching and learning is that promoting connected knowing and deep learning approaches might simultaneously help students develop more sophisticated ways of engaging with the calculators for mathematical inquiry and exploration. Conversely, teaching that promotes separate knowing and surface approaches (particularly when there is an overly strong focus on examination success) might encourage students to use calculators at a lower level, by blindly following instructions and procedures, and/or replacing pen-paper computations.

Additionally, there is a gender difference between students' ways of knowing and learning mathematics, with males higher on Connected Knowing-Deep Approach than females, and females higher on Separate Knowing-Surface Approach. Given that these beliefs and approaches are found to be associated with students' ways of engaging with the calculator and could potentially influence students' mathematics achievement, further study is needed to investigate and address the gender difference within the two regional contexts.

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